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Local Buckling of Honeycomb Sandwich Plates Under Action of Transverse Shear Forces

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In this paper, we evaluate the critical transverse shear forces for the local buckling of honeycomb sandwich plates subjected to lateral loads. The evaluation is based on the stress field accounting for the stresses on the microscale in the honeycomb's hexagonal cells. These microscale stresses are computed by the two-scale method of homogenization theory for periodic media. The elastic restraints resulting from neighboring walls of hexagons are taken into account. The critical shear forces presented here can be used as a design criterion for sandwich plates with honeycomb cores.

Introduction

BECAUSE light material is used for the core, a sandwich plate can have a strong flexural rigidity and a high strength-to-weight ratio. Consequently, the sandwich plate is an efficient structure. One of the most widely used core materials is the honeycomb cellular structure. Because of the nature of thin walled structures, the honeycomb core may collapse because of the local instability of its hexagon cells. Although buckling of sandwich plates and local instability of honeycomb cores under the loads in the plate plane have been studied,[1-3] the local buckling of honeycomb cores under lateral loads has not received similar attention from researchers. Since a lateral load is a common loading condition, the critical behavior of honeycomb cores of sandwich plates subjected to lateral loads is an important criterion for the analysis and design of honeycomb sandwich plates. The objective of this study is to evaluate the critical transverse shear forces for the local buckling of honeycomb sandwich plates under the action of transverse shear forces.

The accuracy of stress distribution in honeycomb cellular structures is essential for the local buckling analysis of honeycomb cores. Basically, a honeycomb core is an inhomogeneous medium. For simplicity, the equivalent mechanical properties of homogenized cores are often used in the general analysis of honeycomb sandwich plates.[4] A honeycomb cellular structure has two spatial scales, that is, the scale of a hexagonal cell, termed the microscale here, and the scale of the core, termed the macroscale. Corresponding to the two spatial scales, the stresses in a honeycomb core can also be divided into two scales, i.e., the stresses on the microscale, which are the local solution of the original cellular structure, and the ones on the macroscale, which are the global solution obtained from the equivalent properties of homogenized cores. For the global response, only the macroscale solution is required. However, the local buckling of a honeycomb's hexagon cells is related to the stresses on the microscale. Therefore, the microscale solution must be solved. It should be pointed out that the conventional approaches of structural analysis are not capable of determining the stresses on the microscale. The theory of homogenization[5] is a rigorous method for evaluating the equivalent material properties of periodic media. The two-scale method of homogenization theory[6] was used in the authors' previous papers to derive the equivalent mechanical properties of honeycombs.[7,8] The two-scale method is employed again in this work to evaluate the stresses on the microscale after the macroscale solution of the homogenized cores has been solved.

The critical behavior of a plate strongly depends on its boundary condition. Every wall of a honeycomb's hexagon cells, termed a panel here, is supported by its neighboring panels, which provide elastic restraints to the panel under consideration. In this study, the elastic restraints at panel edges are modeled by their equivalent rotational springs. Based on the critical shear stresses for plates with simply supported and clamped edges,[9] a simple scheme is proposed for the evaluation of the critical shear stress of elastically restrained plates.

Since the stresses on the microscale of honeycomb cellular structures are taken into account in this work, the resulting solution for the local buckling analysis of honeycomb cores is accurate. Therefore, the present solution provides a better criterion for the design of honeycomb sandwich plates subjected to lateral loads.

Shear Stress on Microscale in a Honeycomb Core

In this study, it is assumed that a honeycomb core is a cellular structure, which is built periodically or nearly periodically from certain basic substructures, called basic cell here. The scale of the honeycomb is referred to as the macroscale, and the scale of the basic cell is termed the microscale. The geometry of a typical hexagon cell of a honeycomb core is depicted in Fig. 1. For a honeycomb sandwich plate subjected to a lateral load, each panel of the hexagonal core only has shear stress and normal stress in the z direction of the plate. As in the bending of homogeneous plates, the normal stress in the

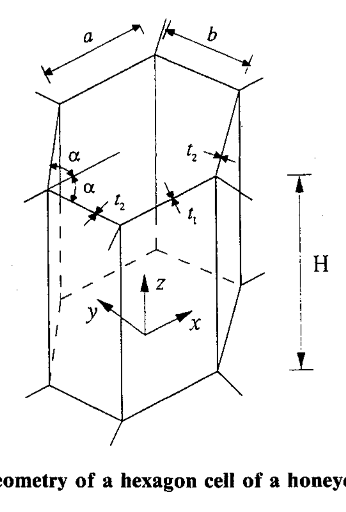

Fig. 1 Geometry of a hexagon cell of a honeycomb core.

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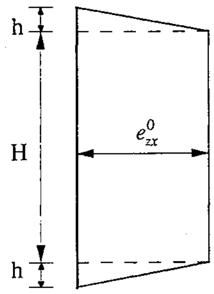

Fig. 2 Shear strain distribution in a homogenized sandwich plate with a deep core.

z direction of the honeycomb core of a bending plate is much smaller than its transverse shear stress. Therefore, the major concern in a honeycomb core is its shear stress.

Corresponding to the two spatial scales, the stresses in a honeycomb core can be divided into two levels, i.e., stresses on the macroscale and on the microscale, respectively. The stresses on the macroscale are the global solution of the homogenized plate, and the stresses on the microscale are the local solution of the original inhomogeneous plate. The honeycomb core can be homogenized by an averaging process over the basic cell on the microscale, and the equivalent mechanical properties of the homogenized core are used for the global solution. However, the stresses on the microscale must be considered in the local buckling analysis of honeycomb cores.

For most sandwich plates with honeycomb cores used in engineering applications, the depth of the core is much larger than both the thickness of the face panel and the dimensions of its hexagonal cells, i.e., $h/H \ll 1$, $a/H \ll 1$, and $b/H \ll 1$ as shown in Figs. 1 and 2. For simplicity, only honeycomb cores with such geometric aspects are considered in this work.

By using the equivalent material properties, the transverse shear forces, Q_x and Q_y, of a homogenized honeycomb sandwich plate, which are the global solution, can be obtained from the standard methods of structural analysis. For the sandwich plate with a deep honeycomb core, the core can be treated as an antiplane one. Consequently, the macro shear strains in such a core are uniformly distributed across its depth as shown in Fig. 2.[1] Let G_{zx} and G_{zy} be the principal equivalent transverse shear moduli of a honeycomb core; then the corresponding shear strains of the homogenized honeycomb core on the macroscale take the form

$$2e_{zx}^0 = \frac{Q_x}{G_{zx}(H+h)} \tag{1}$$

$$2e_{zy}^0 = \frac{Q_y}{G_{zy}(H+h)} \tag{2}$$

where H is the depth of the honeycomb core, h is the thickness of the face panel, and the superscript 0 signifies the solution on the macroscale. The general forms of G_{zx} and G_{zy} obtained from a two-dimensional basic cell are presented in Ref. 7. The three-dimensional solution for G_{zx} and G_{zy} of a regular honeycomb core is given in Ref. 8.

The macrostrains in hexagon cells can be evaluated from the standard strain transformation. If one lets w' be the microdeflection in the z direction and u_ξ' be the microdisplacement along the local axis ξ of a panel, then the perturbation solution of shear strain on the microscale in the panel local axis $e_{z\xi}'$ can be expressed as

$$2e_{z\xi}' = \frac{\partial w'}{\partial \xi} + \frac{\partial u_\xi'}{\partial z} \tag{3}$$

According to their orientations, the panels in a honeycomb core can be divided into two types, i.e., the panels parallel to the x axis and the panels oblique to it. From now on, the subscript zx is used to represent panels parallel to the x axis, and the subscript $z\xi$ signifies the oblique panels. By using the two-scale method of homogenization and a two-dimensional model, the authors analytically derived the first-order shear strains on the microscale as[7]

$$2e_{zx}' = \frac{\partial w'}{\partial x} = -s2e_{zx}^0 \tag{4}$$

$$2e_{z\xi}' = \frac{\partial w'}{\partial \xi} = s2e_{zx}^0 \tag{5}$$

with

$$s = \frac{t_1 - 2t_2 \cos\alpha}{t_1 + 2t_2 \cdot a/b} \tag{6}$$

The definitions of a, b, t_1, t_2, and α are graphically given in Fig. 1. Equations (4-6) indicate that the micro shear strains in a honeycomb structure depend on the honeycomb's geometry and can be expressed in terms of macrostrain e_{zx}^0. For the regular honeycomb where $a = b$, $\alpha = 60$ deg, and $t_1 = 2t_2$, Eq. (6) gives $s = 1/4$.

A two-dimensional model implies that all quantities are independent of the z direction; for example, $\partial u_\xi'/\partial z = 0$. The numerical study of a three-dimensional basic cell[8] shows that $\partial u_\xi'/\partial z \neq 0$, and the contribution of $u_\xi'(z)$ to $e_{z\xi}'$ depends on the geometry of sandwich plates. The component of shear strain given by $\partial w'/\partial \xi$, strictly speaking, is not constant in the z direction in a three-dimensional model analysis. Nevertheless, except for the area near the face panels, $\partial w'/\partial \xi$ is quite close to the value given by the two-dimensional analytical derivation. Therefore, $\partial w'/\partial \xi$ obtained from the two-dimensional model still can be used in the three-dimensional analysis. The general form of a hexagon cell's shear strains accounting for the strains on the microscale takes the form[7,8]

$$2e_{zx} = 2e_{zx}^0 + 2e_{zx}' = 2e_{zx}^0 - s2e_{zx}^0 + \frac{\partial u_x'}{\partial z} \tag{7}$$

$$2e_{z\xi} = 2e_{z\xi}^0 + 2e_{z\xi}' = \cos\alpha 2e_{zx}^0 + \sin\alpha 2e_{zy}^0 + s2e_{zx}^0 + \frac{\partial u_\xi'}{\partial z} \tag{8}$$

in which $\partial u_x'/\partial z$ and $\partial u_\xi'/\partial z$ can be evaluated numerically for the given geometry of sandwich plates. When honeycomb cellular structures are symmetric about the x and the y axes, $\partial u_x'/\partial z$ and $\partial u_\xi'/\partial z$ are dependent only on e_{zx}^0 and can be expressed in a general form as

$$\frac{\partial u_x'}{\partial z} = \beta 2e_{zx}^0 \tag{9}$$

$$\frac{\partial u_\xi'}{\partial z} = \cos\alpha \frac{\partial u_x'}{\partial z} \tag{10}$$

Consequently, Eqs. (7) and (8) can be rewritten as

$$2e_{zx} = (1 - s + \beta)2e_{zx}^0 \tag{11}$$

$$2e_{z\xi} = (\cos\alpha + s + \beta\cos\alpha)2e_{zx}^0 + \sin\alpha\, 2e_{zy}^0 \tag{12}$$

For a honeycomb made of isotropic material with shear modulus G, the shear stresses accounting for microstrains are of the form

$$\sigma_{zx} = G2e_{zx} = (1 - s + \beta)\frac{G}{G_{zx}(H+h)} Q_x \tag{13}$$

$$\sigma_{z\xi} = G2e_{z\xi} = (\cos\alpha + s + \beta\cos\alpha)\frac{G}{G_{zx}(H+h)} Q_x + \sin\alpha \frac{G}{G_{zy}(H+h)} Q_y \tag{14}$$

By recalling Eq. (6), one can see from the previous equations that shear strains on the microscale have a significant contribution to the corresponding shear stresses. Consequently, in the analysis of a honeycomb core's local buckling it is important to include the microstrains in the shear stresses of hexagon cells.

Analysis Model for Buckling of Hexagon Cells

The local buckling of a honeycomb core is attributable to the instability of any individual panel built-up hexagon cells of the honeycomb. All panels of a honeycomb core are rectangular in shape. Because the depth of a honeycomb core is, in general, much larger than the dimension of its hexagon cells, the local buckling of a honeycomb core subjected to transverse shear forces can be modeled as the buckling of long rectangular plates under the action of shear forces. As mentioned earlier, the stresses accounting for the microstrains should be used for the stress state in the buckling analysis of hexagon cells. Every panel of hexagon cells in a honeycomb core is elastically restrained by its neighboring panels. Such elastic support can be represented by the equivalent rotational springs at the panel edges. The computational model used to evaluate the stiffness of a rotational spring is depicted in Fig. 3. Since the neighboring panels are also connected to other panels, a rotational spring is used at each support for the elastically restrained end. By using the stiffness matrix of a beam element with rotational springs at its ends presented by Shi and Atluri,[10] the rotation at the crown point Φ_A, which corresponds to a moment M_A at the same point, can be calculated. The equivalent rotational stiffness of the elastically restrained support at the point can be defined as

$$k = \frac{M_A}{\Phi_A}, \qquad 0 \le k < \infty \tag{15}$$

It was shown that the critical behavior of a long rectangular plate under shear stresses is insensitive to the boundary conditions along its shorter edges. For example, when the aspect ratio of length to width is larger than 2, the critical shear stress of the plate where two longer edges are clamped and two shorter edges are simply supported is almost equal to that of the plate with all edges clamped. Therefore, there is no need to

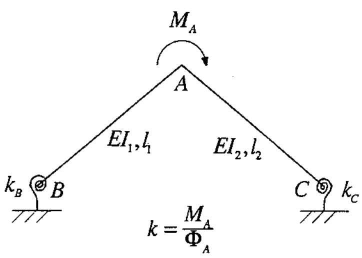

Fig. 3 Computational model for determining rotational spring stiffness.

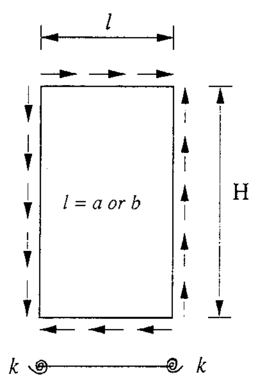

Fig. 4 Analysis model for the buckling of a honeycomb's hexagon cells.

concern the boundary conditions along the shorter edges. The analysis model for the local buckling of a honeycomb core is illustrated in Fig. 4. From the derivation of the critical shear stress of simply supported and clamped plates,[9] it can be conceived that the analytical derivation of buckling shear stresses of elastically restrained plates is very cumbersome. But the same problem can be easily solved by numerical methods, and the numerical solution can be adopted.

When the boundary conditions are changed from simple supports to built-in ends, the corresponding buckling load of columns increases three times, whereas the corresponding critical shear stress of plates increases less than 70%. Therefore, the buckling behavior of plates is much less sensitive to their boundary conditions than that of columns. For convenience, this work attempts to construct an approximate solution for the critical shear stress of elastically supported plates from the analytical solutions for the simply supported and clamped plates.

Notice the analogy on boundary conditions between the plate shown in Fig. 4 and the column with rotational springs; a characteristic equation of the equivalent column for the plate depicted in Fig. 4 can be defined as[9]

$$\frac{\tan u}{u} = -\frac{2EI}{k \cdot l}, \qquad 0 \le k < \infty \tag{16}$$

where k is the stiffness of the rotational spring; and EI and l are, respectively, the flexural stiffness and the width of the panel under consideration. When $k = 0$, which corresponds to a simply supported column, $u = \pi/2$. When $k = \infty$, which corresponds to a clamped column, $u = \pi$. Let τ_{cr}^s and τ_{cr}^c be the critical shear stresses for simply supported and clamped plates, respectively; then a simple and natural interpolation for the critical shear stress of elastically restrained plates τ_{cr}^e takes the form

$$\tau_{cr}^e = (1 - \zeta)\tau_{cr}^s + \zeta\tau_{cr}^c, \qquad 0 \le \zeta \le 1 \tag{17}$$

in which ζ is a nondimensional parameter. From the buckling of a column with elastically restrained ends, ζ can be defined as

$$\zeta = \left(\frac{u - \pi/2}{\pi/2}\right)^2 = \left(\frac{2u}{\pi} - 1\right)^2 \tag{18}$$

for $\pi/2 \le u \le \pi$ and $0 \le \zeta \le 1$.

The values of τ_{cr}^s and τ_{cr}^c corresponding to different aspect ratios can be found in literature.[9] When a rectangular plate is long enough, say $H/l > 4$, its critical shear stress is converging to a constant.[9] Because the depth of a honeycomb core is much larger than the dimension of hexagon cells, the results of τ_{cr}^s and τ_{cr}^c for the infinitely long plate[9] could be used in Eq. (17). Accordingly, Eq. (17) becomes

$$\tau_{cr}^e = [5.35(1 - \zeta) + 8.99\zeta]\frac{\pi^2 EI}{l^2 t} = \lambda\frac{\pi^2 EI}{l^2 t} \tag{19}$$

in which t is the panel thickness under consideration, and λ is the critical shear stress parameter defined as

$$\lambda = 5.35(1 - \zeta) + 8.99\zeta \tag{20}$$

By equalizing the shear stress in a panel to the critical shear stress, one obtains the critical transverse shear forces for the panels parallel to the x axis and the panels oblique to the x axis, respectively, as

$$Q_x = \frac{G_{zx}(H + h)}{G(1 - s + \beta)}\frac{\pi^2 t_1^2 E}{12(1 - \nu^2)a^2}\lambda_x \tag{21}$$

$$(\cos\alpha + s + \beta\cos\alpha)Q_x + \frac{G_{zx}}{G_{zy}}\sin\alpha\, Q_y = \frac{G_{zx}}{G}(H + h)\frac{\pi^2 t_1^2 E}{12(1 - \nu^2)a^2}\left(\frac{t_2}{t_1}\right)^2\left(\frac{a}{b}\right)^2\lambda_\xi \tag{22}$$

where λ_x and λ_ξ are the critical shear stress parameters defined in Eq. (20) for the parallel and oblique panels, respectively.

In general, G_{zx} and G_{zy} can be expressed in terms of G as[7]

$$G_{zx} = \mu_x \frac{t_1}{a} G \tag{23}$$

$$G_{zy} = \mu_y \frac{t_1}{a} G \tag{24}$$

where μ_x and μ_y are nondimensional parameters on the order of unity. Consequently, Eqs. (21) and (22) become

$$Q_x = \mu_x \frac{H + h}{1 - s + \beta} \frac{\pi^2 E}{12(1 - \nu^2)} \left(\frac{t_1}{a}\right)^3 \lambda_x \tag{25}$$

$$(\cos\alpha + s + \beta\cos\alpha)Q_x + \frac{\mu_x}{\mu_y}\sin\alpha\, Q_y$$
$$= \mu_x(H + h)\frac{\pi^2 E}{12(1 - \nu^2)}\left(\frac{t_1}{a}\right)^3\left(\frac{t_2}{t_1}\right)^2\left(\frac{a}{b}\right)^2\lambda_\xi \tag{26}$$

When $\alpha \neq 90$ deg, Eq. (26) can be rewritten as

$$Q_x + \frac{\mu_x}{\mu_y}\frac{1}{\cos\alpha + s + \beta\cos\alpha}\sin\alpha\, Q_y = \mu_x \frac{H+h}{\cos\alpha + s + \beta\cos\alpha}$$
$$\times \frac{\pi^2 E}{12(1 - \nu^2)}\left(\frac{t_1}{a}\right)^3\left(\frac{t_2}{t_1}\right)^2\left(\frac{a}{b}\right)^2\lambda_\xi \tag{26a}$$

The critical condition for the local buckling of honeycomb cores is governed by the smaller value between Q_x given by Eq. (25) and the combination of Q_x and Q_y obtained from Eq. (26). The previous equations show that the critical transverse shear forces depend on both the geometry of honeycomb structures and the loading condition, primarily the ratio of Q_x to Q_y. For a given honeycomb core and loading condition, the dominant equations between Eqs. (25) and (26) can be easily identified. This will be illustrated in the next section.

Local Buckling of Regular Honeycomb Core

Regular honeycomb cellular structures, in which $a = b$ and $\alpha = 60$ deg, are the most widely used honeycomb cores. With respect to the thickness of a honeycomb's hexagon cells, the regular honeycombs can be further divided into two types, i.e., a uniform thickness honeycomb where $t_1 = t_2 = t$ and a nonuniform thickness one where $t_1 = 2t_2 = 2t$. By using the results presented in the previous section, this section illustrates the evaluation of the critical transverse shear forces of the two types of regular honeycombs.

Type 1: $a = b$, $\alpha = 60$ deg, and $t_1 = t_2 = t$

Using the results given in the authors' previous papers,[7,8] one has

$$\mu_x = \mu_y = \mu = \frac{1}{\sqrt{3}}$$
$$s = \beta = 0$$

Because in this case all panels have the same supporting condition, it follows

$$\lambda_x = \lambda_\xi = \lambda$$

The parameter λ defined in Eq. (20) is a function of the rotational spring stiffness k defined in Eq. (15). It can be easily shown that λ is insensitive to a small change on k. When the structure depicted in Fig. 3 is treated as a simply supported toggle, the equivalent rotational stiffness of the elastically restrained ends takes the value of $k = 6EI/l$. Solving Eqs. (16), (18), and (20) in turn, one obtains

$$u = 2.46$$
$$\zeta = 0.32 \tag{27}$$
$$\lambda = 6.51$$

It is worth mentioning that for a plate with aspect ratio $H/a = 5$ the finite element buckling analysis of this problem gives $\lambda = 6.47$ where a 12×24 mesh is used. Therefore, the linear interpolation given in Eqs. (18–20) is a quite good approximation for the critical shear stresses of plates with rotationally elastic supports.

A substitution of the parameters in Eq. (27) into Eqs. (25) and (26) leads to

$$Q_x = 3.76(H + h)\frac{\pi^2 E}{12(1 - \nu^2)}\left(\frac{t}{a}\right)^3 \tag{28}$$

$$\frac{1}{2}Q_x + \frac{\sqrt{3}}{2}Q_y = 3.76(H + h)\frac{\pi^2 E}{12(1 - \nu^2)}\left(\frac{t}{a}\right)^3 \tag{29}$$

Suppose $Q_x \neq 0$ and define

$$p = \frac{Q_y}{Q_x} \tag{30}$$

then Eq. (29) can be rewritten as

$$\frac{1}{2}(1 + \sqrt{3}p)Q_x = 3.76(H + h)\frac{\pi^2 E}{12(1 - \nu^2)}\left(\frac{t}{a}\right)^3 \tag{31}$$

It follows from Eqs. (28) and (31) that when $p < 1/\sqrt{3}$ the instability of the honeycomb core is caused by the buckling of parallel panels with the critical shear force

$$(Q_x)_{cr} = 3.76(H + h)\frac{\pi^2 E}{12(1 - \nu^2)}\left(\frac{t}{a}\right)^3 \tag{32}$$

for $Q_y \leq Q_x/\sqrt{3}$. When $Q_x \neq 0$ and $p > 1/\sqrt{3}$, the buckling of the honeycomb core is initiated by the buckling of oblique panels with the critical shear force

$$(Q_x)_{cr} = \frac{7.52}{1 + \sqrt{3}p}(H + h)\frac{\pi^2 E}{12(1 - \nu^2)}\left(\frac{t}{a}\right)^3 \tag{33}$$

$$(Q_y)_{cr} = p(Q_x)_{cr} \tag{34}$$

for $Q_y \geq Q_x/\sqrt{3}$. In the case of $Q_y = Q_x/\sqrt{3}$, Eqs. (28) and (29) give the same solution shown in Eqs. (32) and (33), which means that all panels buckle simultaneously.

When $Q_x = 0$, $(Q_y)_{cr}$ can be obtained from Eq. (26) as

$$(Q_y)_{cr} = 4.34(H + h)\frac{\pi^2 E}{12(1 - \nu^2)}\left(\frac{t}{a}\right)^3 \tag{35}$$

for $Q_x = 0$.

Type 2: $a = b$, $\alpha = 60$ deg, and $t_1 = 2t_2 = 2t$

In this case[7,8]

$$\mu_x = \frac{1.56}{2\sqrt{3}}, \qquad \mu_y = \frac{1}{2\sqrt{3}}$$
$$s = 0.25, \qquad \beta = 0.09 \tag{36}$$
$$\lambda_x = 5.41, \qquad \lambda_\xi = 8.08$$

Because $t_1 = 2t_2$, it is obvious that the right-hand side of Eq. (26a) is always smaller than that of Eq. (25). Consequently, the buckling of a honeycomb core with such a geometry is caused by its oblique panels.

When $Q_x \neq 0$ and $Q_y = pQ_x$, Eq. (26) yields

$$(Q_x)_{cr} = 14.55\,\kappa_x(H + h)\frac{\pi^2 E}{12(1 - \nu^2)}\left(\frac{t}{a}\right)^3 \tag{37}$$

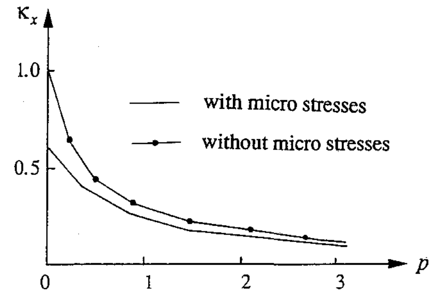

Fig. 5 Coefficient of critical shear force for $(Q_x)_{cr}$.

with

$$\kappa_x = \frac{1}{1.64 + 1.56\sqrt{3}p}$$

When $Q_x = 0$, Eq. (26) gives

$$(Q_y)_{cr} = 5.39(H+h)\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{a}\right)^3 \tag{38}$$

The curve of κ_x vs p is plotted in Fig. 5. The κ_x-p curve for the case in which the strains on the microscale are not considered, marked by dots, is also given in the figure for comparison. When Q_x is a dominant force, the figure shows that the critical force taking into account the microstrains is significantly different from that neglecting the microstrains. Therefore, the consideration of microstrains is important in the analysis of local buckling in honeycomb cores.

Summary and Conclusions

The local buckling of honeycomb sandwich plates subjected to lateral loads is controlled by the instability of a honeycomb's hexagon cells. Through the examination of the critical behavior of hexagon cells, this work has presented the evaluation of critical transverse shear forces for the local buckling of regular honeycomb cores. The stress state in a hexagon panel is evaluated by the two-scale method of homogenization for periodic media, which gives the stresses on the microscale of honeycombs. The elastic supports resulting from the neighboring panels are modeled by rotational springs at the panel's edges. The analysis model for the buckling of a hexagon panel is an elastically restrained plate subjected to a shear stress field. The closed solutions for simply supported and clamped plates under the action of shear stress are used to interpolate the solution for the elastically restrained plate. Because stresses on the microscale are taken into account in this study, the resulting critical transverse shear forces are accurate and provide a better criterion for the design of honeycomb sandwich plates subjected to lateral loads.

The following conclusions can be drawn from the present study.

1) Because of the inhomogeneity of honeycomb cores, the real stresses in the cell's panels, which control the local buckling of honeycomb cores, can be significantly different from the solution given by the analysis of homogenized cores. To obtain an accurate solution, the microstress must be taken into account in the local buckling analysis of honeycomb cores.

2) The two-scale method of homogenization theory is an efficient means for the analysis of honeycomb cores. Not only can it be used for the derivation of equivalent mechanical properties of honeycombs but also for the evaluation of shear stresses on the microscale in hexagon cells.

3) Every panel of a honeycomb's hexagon cells is supported by its neighboring panels, which provide elastic restraints to the panel under consideration. Such elastic restraints must be considered for the boundary condition of the panel's critical behavior.

4) The critical transverse shear forces of a honeycomb sandwich plate depend on both the geometry of the honeycomb core and the ratio of transverse shear forces in different directions.

5) For a regular honeycomb core with $t_1 = 2t_2$, when $Q_y < Q_x/\sqrt{3}$, the local buckling of the core is caused by the instability of honeycomb's parallel panels. When $Q_y > Q_x/\sqrt{3}$, the local buckling of the core is initiated by the honeycomb's oblique panels. When $Q_y = Q_x/\sqrt{3}$, all panels buckle simultaneously.

6) For a regular honeycomb core with $t_1 = 2t_2 = 2t$, the local buckling of the core is always caused by the instability of the honeycomb's oblique panels.

Only the local instability of honeycomb cores is studied here. The postbuckling behavior of honeycomb cores presents a much more difficult problem and will be investigated later.

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